Astronomy Proficiency Exam 2020
Planetary Systems (405)
Aug 21, 2020 — 2:00–5:00 PM

Answer all 5 questions. Each is worth 20 points. Please start your answer to each question on a new page. When possible, derive an algebraic expression first before calculating values.

1. **Orbital motion.** A comet of mass $m = 10^{12}$ kg is observed to have a period of 125 years and an orbital eccentricity of 0.5.

   (a) [**6 pts**] Calculate the perihelion and aphelion distances of the comet (i.e., the closest and farthest distances from the Sun).

   (b) [**6 pts**] Calculate the speeds (in km s^{-1}) at which the comet will be moving at perihelion and at aphelion.

   (c) [**8 pts**] Calculate the instantaneous kinetic energy change at aphelion needed to move the comet into a circular orbit at the aphelion distance. Perform the same calculation for perihelion and compare your results.

2. **Interior Structure.** Suppose that Mercury, with a radius of R=2400 km, has an iron core of uniform density 8000 kg m^{-3} surrounded by a rocky mantle of uniform density 3000 kg m^{-3}. We measure the average density of Mercury to be 5400 kg m^{-3}.

   (a) [**6 pts**] By writing down expressions for the masses of the core, mantle, and planet as a whole, determine the fraction of the total volume of the planet occupied by the core.

   (b) [**5 pts**] Determine the fraction of the total mass of the planet contained within the core.

   (c) [**9 pts**] Calculate the moment of inertia ratio I/MR^2 for Mercury assuming spherical symmetry, and compare it with the expected value for a constant density sphere. (Hint: Recall that I obeys the principle of superposition.)

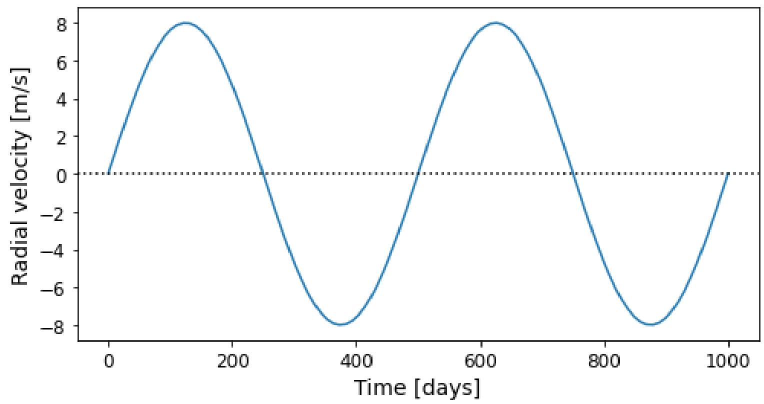
3. **Making the Earth.** To come up with the Minimum Mass Solar Nebula, we spread out the mass of the planets into annuli until they begin to overlap.

   (a) [**6 pts**] Suppose we do this exercise for the Earth: spread one Earth mass of solids uniformly between 0.85 and 1.25 AU. What mass surface density do you end up with, in kg m^{-2}? If the vertical thickness of the annulus is 0.01 AU, what is the average mass density, in kg m^{-3}?

   (b) [**4 pts**] Suppose we divide up the mass contained in the annulus into spherical asteroids 1 km in radius, each with a density of 3000 kg m^{-3}. How many asteroids are needed to equal the mass of the Earth?

   (c) [**10 pts**] If the asteroids are spread uniformly throughout the volume of the annulus, calculate the mean free path ℓ for collisions for an object moving through the annulus. (For simplicity you may ignore the orbital motion of the asteroids.) Discuss the likelihood of a collision for an object traveling a distance equal to the circumference of the Earth's orbit.

4. **Extrasolar Planet.** A star with a mass of 0.6 $M_\odot$ is observed to undergo a periodic variation in radial velocity with a period of 500 days (see figure). The maximum speed of the star towards or away from Earth is 8 m s^{-1}.



(a) **[6 pts]** Calculate the orbital semi-major axis of the planet in AU.

(b) **[6 pts]** Assume the system is viewed at an inclination of 30° from face-on. If you were able to observe the *planet's* radial velocity curve, what would be its amplitude in m s^{-1}?

(c) **[8 pts]** Given this inclination, what is the planet's mass in Earth masses? How important is the orbital inclination for determining whether this is a terrestrial or giant planet?

5. **Exoplanet Atmosphere.** An M-type star with a radius of 0.25 $R_\odot$ is transited by a planet which produces a transit depth of $\delta F/F = 5 \times 10^{-3}$.

(a) **[7 pts]** Calculate the radius of the planet, R_p, in km. If the mean density of the planet is taken to be 3000 kg m^{-3}, also calculate its mass in Earth masses.

(b) **[6 pts]** If the planet's equilibrium temperature is 250 K, will its gravity be sufficient to retain an H_2 atmosphere?

(c) **[7 pts]** Assuming H_2 is present, estimate the scale height of H_2 in the atmosphere of the planet. Discuss whether the thin atmosphere assumption is valid.