Answer all 5 questions. Each is worth 20 points. Please start your answer to each question on a new page. When possible, derive an algebraic expression first before calculating values.

1. **Orbital motion.** A comet of mass \( m = 10^{12} \) kg is observed to have a period of 125 years and an orbital eccentricity of 0.5.

   (a) [6 pts] Calculate the perihelion and aphelion distances of the comet (i.e., the closest and farthest distances from the Sun).
   
   (b) [6 pts] Calculate the speeds (in km s\(^{-1}\)) at which the comet will be moving at perihelion and at aphelion.
   
   (c) [8 pts] Calculate the instantaneous kinetic energy change at aphelion needed to move the comet into a circular orbit at the aphelion distance. Perform the same calculation for perihelion and compare your results.

2. **Interior Structure.** Suppose that Mercury, with a radius of \( R = 2400 \) km, has an iron core of uniform density 8000 kg m\(^{-3}\) surrounded by a rocky mantle of uniform density 3000 kg m\(^{-3}\). We measure the average density of Mercury to be 5400 kg m\(^{-3}\).

   (a) [6 pts] By writing down expressions for the masses of the core, mantle, and planet as a whole, determine the fraction of the total volume of the planet occupied by the core.
   
   (b) [5 pts] Determine the fraction of the total mass of the planet contained within the core.
   
   (c) [9 pts] Calculate the moment of inertia ratio \( I/MR^2 \) for Mercury assuming spherical symmetry, and compare it with the expected value for a constant density sphere. (Hint: Recall that \( I \) obeys the principle of superposition.)

3. **Making the Earth.** To come up with the Minimum Mass Solar Nebula, we spread out the mass of the planets into annuli until they begin to overlap.

   (a) [6 pts] Suppose we do this exercise for the Earth: spread one Earth mass of solids uniformly between 0.85 and 1.25 AU. What mass surface density do you end up with, in kg m\(^{-2}\)? If the vertical thickness of the annulus is 0.01 AU, what is the average mass density, in kg m\(^{-3}\)?

   (b) [4 pts] Suppose we divide up the mass contained in the annulus into spherical asteroids 1 km in radius, each with a density of 3000 kg m\(^{-3}\). How many asteroids are needed to equal the mass of the Earth?

   (c) [10 pts] If the asteroids are spread uniformly throughout the volume of the annulus, calculate the mean free path \( \ell \) for collisions for an object moving through the annulus. (For simplicity you may ignore the orbital motion of the asteroids.) Discuss the likelihood of a collision for an object traveling a distance equal to the circumference of the Earth’s orbit.
4. **Extrasolar Planet.** A star with a mass of 0.6 $M_\odot$ is observed to undergo a periodic variation in radial velocity with a period of 500 days (see figure). The maximum speed of the star towards or away from Earth is 8 m s$^{-1}$.

(a) [6 pts] Calculate the orbital semi-major axis of the planet in AU.

(b) [6 pts] Assume the system is viewed at an inclination of 30° from face-on. If you were able to observe the planet’s radial velocity curve, what would be its amplitude in m s$^{-1}$?

(c) [8 pts] Given this inclination, what is the planet’s mass in Earth masses? How important is the orbital inclination for determining whether this is a terrestrial or giant planet?

5. **Exoplanet Atmosphere.** An M-type star with a radius of 0.25 $R_\odot$ is transited by a planet which produces a transit depth of $\delta F/F = 5 \times 10^{-3}$.

(a) [7 pts] Calculate the radius of the planet, $R_p$, in km. If the mean density of the planet is taken to be 3000 kg m$^{-3}$, also calculate its mass in Earth masses.

(b) [6 pts] If the planet’s equilibrium temperature is 250 K, will its gravity be sufficient to retain an H$_2$ atmosphere?

(c) [7 pts] Assuming H$_2$ is present, estimate the scale height of H$_2$ in the atmosphere of the planet. Discuss whether the thin atmosphere assumption is valid.