Dispersion in Financing Costs and Development

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Very preliminary. Please do not circulate.

Abstract

We study how dispersion in financing cost and financial contract enforcement affect entrepreneurship, firm dynamics and productivity. We use employee-employer administrative linked data combined with data on financial transactions of all formal firms in Brazil to show how interest rate spreads vary with firm size, age, among other characteristics. We present a general equilibrium model with endogenous occupational choice based on a modified version of Buera, Kaboski, and Shin (2011), which are consistent with those facts of the the credit market. We then provide evidence on the allocative effects of financial reforms. Eliminating dispersion in financing cost leads to more credit and higher output due to cheaper credit for productive agents with low assets. In addition, abstracting from heterogeneity in interest rate spreads understates the impacts of financial reforms that improve the enforcement of credit contracts.

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1 Introduction

One of the striking features of the credit market is the sizable gap between lending and deposit rates. For instance, according to the International Financial Statistics, the average interest rate spread (lending minus deposit rate) is approximately 0.7 percent in Japan, 3 percent in the United States, 5 percent in Italy, 10 percent in Uruguay and 40 percent in Brazil. Banerjee (2003) also shows that there is extreme variability in the interest rate charged by lender for similar loan transactions within the same economy,[1] such that richer entrepreneurs borrow more and pay lower rates of interest. In Section 2, we use the Brazilian Public Credit Register[2] (CIS - Credit Information System, managed by Central Bank of Brazil) and combine it with Brazil's linked employer-employee administrative data set (RAIS - Relação Anual de Informações Sociais) to understand how loan interest rates and size vary depending on firm characteristics. We show that even controlling for loan type, loan maturity, a credit risk index and sectorial fixed effects and location fixed effects, the loan interest rate and the volume of credit vary considerably with firm characteristics, such as firm size and age. In particular, young and small firms pay higher loan interest rate than old and large firms. For instance, a firm with 300 employers will pay in interest rates approximately 3 percentage points less than a firm with 30 employers and 5.5 percentage points less than a firm with 3 employers, even controlling for loan size, loan maturity, firm age, sector of production and other characteristics.

In this paper, we investigate the effects of dispersion in the cost of financial intermediation on entrepreneurship, firm dynamics and economic development in an economy in which financial contracts are imperfectly enforced. We build a general equilibrium model with heterogeneous agents and endogenous occupational choice similar to the one of Buera, Kaboski, and Shin (2011). In each time period, agents can choose to be either a worker or an entrepreneurs, as in Lucas (1978). Agents are heterogenous in their ability as entrepreneurs and in each period they face a positive probability of drawing a new entrepreneurial ability from an invariant Pareto distribution. This endogenously generates firm entry and exit, as in Hopenhayn (1992). The occupational decision of the agents is also restricted by their initial wealth

[1] See also Banerjee and Duflo (2010) and Gilchrist, Sim, and Zakrajsek (2013).

[2] This is a confidential loan level data set, which covers all the credit operations in Brazil since January 2004 and contains information on loan amount, loan type, loan maturity and interest rates.

and two credit market frictions. First of all, financial intermediaries are competing to lend to each entrepreneur and their monitoring technology is decreasing in the entrepreneur's net worth, such that competition implies that loan interest rates will be decreasing in the entrepreneur's initial asset level. This endogenously generates a dispersion in the interest rate spread by firm size. In addition, there is limited enforcement of financial contracts and borrowers who renege their obligations face a cost proportional to their output net of wage payments. Then the capital of each entrepreneur depends on her net worth and project profitability. Therefore, both the loan size and the loan interest rate are jointly determined, which is a fact emphasized by Banerjee (2003) in his characterisation of credit markets in developing countries.

We calibrate and estimate the model to be consistent with key firm level characteristics of the Brazilian economy, such as firm size, exit and entry rates. We also endogenously match the relationship between loan interest rate spread and firm size, as observed in the Brazilian micro level evidence. We then study how financial frictions affect firm dynamics, such as firm growth, entry and exit. In addition, we investigate the quantitative aggregate effects of the two financial frictions. We show that they produce very different aggregate effects. Enforcement of financial contracts has a larger aggregate impact on output (for a similar increase in the level of credit) than intermediation costs. This is because when the enforcement of financial contracts increases then entrepreneurs can borrow more for a given interest rate, and this affects mainly more productive entrepreneurs who are credit constrained and can now grow at a faster rate. When intermediation costs decrease, then this affects all entrepreneurs who are borrowing and those who can now borrow at a lower rate. This also increases production but the credit is not mainly allocated to the most productive entrepreneurs who are constrained.

This paper is related to a large literature on the effects of financial frictions on entrepreneurship and economic development, such as Antunes, Cavalcanti, and Villamil (2008), Banerjee and Moll (2010), Buera, Kaboski, and Shin (2011), Buera and Shin (2013), Erosa (2001), Greenwood, Sanchez, and Wang (2010), Midrigan and Xu (2014), Moll (2014), among others. We differ from these papers in these two following ways. First of all, most of the papers in this literature either consider only one type of financial friction and when there is a spread between the loan and the deposit rate such spread does not vary within the same economy. The only exception is Greenwood, Sanchez, and Wang (2010). They model the financial contract in detail and show how the monitoring technology can endogenously generate dispersion in interest rates. However, they abstract from self-financing, which is a key feature in our modeling strategy. In addition, we use Brazilian administrative firm level data combined with data on financial transactions of firms to endogenously estimate model parameters that are consistent with firm dynamics and loan characteristics observed in the Brazilian economy.

2 Empirical Analysis

This section provides the empirical evidence on dispersion in financing costs that provides motivation, informs our modeling strategies, and ultimately disciplines our quantitative approach.

2.1 Data

We first describe the two main datasets used in the empirical analysis. Data on bank loans to all formal firms in Brazil are from a large repository of corporate loan contracts coming from the Brazilian Public Credit Register (SCR - *Sistema de Informação de Crédito*). This is a confidential loan level database protected by Brazilian banking privacy law, owned and managed by the Central Bank of Brazil. It provides detailed information on all loans granted from January 2005 until December 2016. For any lbank-to-firm loan during the period of analysis, we identify the lender, borrower, size of the loan, the interest rate on the loan, the loan maturity, default rates, the currency denomination of the loan and whether or not it was an earmarked (i.e., subsidized) rate.[3] The dataset allows us to collect information on the borrower-lender relationship, such as the length of a firm-bank relationship. We calculate the firms' outstanding loans spreads by the difference between the weighted average of firms' outstanding loans rates and the one-year interbank deposit rate.

The other main dataset that we use in our empirical analysis is RAIS (*Relação Anual de Informações Sociais*), which is a matched employer-employee administrative dataset covering all formally registered firms in Brazil. This

[3]The Brazilian Development Bank (BNDES) is the main financing institution for productive investment in the country, and it offers subsidized interest rates for long-term investments.

a mandatory annual survey maintained by the Ministry of Labor and Employment of Brazil. RAIS provides detailed information on firms, such as economic sector of activity and location. It also contains information on employees, such as age, gender and education; and on employment relationship, such as wage, tenure, hiring and layoff dates. It is also possible to identify the date of entry and exit of firms. With these data, we can capture important firm dynamics for all formal firms in Brazil, such as entry, exit and growth by the number of employees and by the wage bill of the firm.

Using the unique firm tax identifier, we merge the SCR and RAIS datasets. Table 12 in Appendix A provides descriptive statistics for the main variables used in our empirical analysis. The definition and source of all variables are also contained in Appendix A. There are approximately 12 millions observations in the sample. Within this sample, the average size of a formal firm in Brazil is 21 employees and the median size is 4 employees. About 4 percent of the loans are non-performing and the average time of the firm-bank relationship is roughly 80 months or approximately 6.6 years.

2.2 Empirics

We start with some basic summary statistics on the interest rate spreads, which are high and variable. Firms are on average paying a sizeable spread: relative to the one-year interbank deposit rate, the average spread in the sample is 44 percentage points and the median spread is about 23 percentage points. There is also large variability in the spread rate since the standard deviation of the spread rate is 60 percentage points. Some firms are paying a loan interest rate that is above 100 percentage points in excess to the one-year interbank deposit rate, while other firms with access to earmarked credit provided, often but not exclusively, by the BNDES are paying a negative spread. **XXinsert the average loan maturity**

The variation in spreads follows discernible patterns. Table 1 reports the Ordinary Least Square (OLS) regression in which the dependent variable is the logarithm of one plus the spread rate and the regressors are the set of controls. These controls include loan and borrower characteristics, such as maturity, earmarked credit, firm-bank relationship in months, firm-level controls (e.g., sector of activity, location), and time. There are also controls for proxies for the risk of default of the loan, such as the contemporaneous rate of non-performing loans and the lead and lag of this variable. mentioned above. As we can easily observe, there is a positive relationship between the interest rate spread and the rate of non-performing loans of a firm, and a negative correlation between the interest rate spread and the maturity of the loan. In addition, the higher is the number of banks from which the firm is borrowing, the lower is the spread on its outstanding loans, and the longer is the firm-bank relationship then the lower is the spread.

Nevertheless, much of the variation in the spread cannot be explained by observed factors. Approximately 12 percent of the observed variation in the spread is explained by the loan characteristics presented in the first half of table 1, including the rate of non-performing loans at the firm level and the lag and lead of this variable. The introduction of loan type controls increases the explanation of the observed variability in the spreads from 12 percent to 45 percent.[4] In our most complete and preferred specification with a full set of different fixed effects (e.g., location, sector, time), i.e. Column (4) of Table 1, approximately 50 percent of the observed variability in the spread at the firm level can be explained by the regressors used in this regression. Thus, roughly half of the variation remains unexplained.

We show this more clearly in Table 2, which presents summary statistics for the dependent variable from Table 1 (the logarithm of one plus the spread rate) and the residuals of all the regressions in Table 1. The analysis of the regression residuals shows that there is still a large variability in the spread after controlling for a list of loan characteristics and a variety of fixed effects. For instance, the standard deviation of the residual of Model (4) of Table 1 is 0.226, which is approximately 70% of the of the unconditional standard deviation of the logarithm of one plus the spread rate, reported in the first column of Table 2. This translates into a standard deviation of the spread rate of approximately 25%.

In order to understand the remaining variability in the spread rate at the firm level, we then regress the residuals of the regressions of Table 1 on firm fixed effects. The summary of such regressions are reported in Table 3. Firm fixed effects contain firm idiosyncratic characteristics, which do not change with time, such as sector of activity. But it can also capture features which hardly change within a ten years period, such as location, but also other intangible characteristics such as some managerial practices (e.g., being a family run business) of the firm. As we can observe, for our most

[4]These controls are the proportion of credit which is in different type of loans, such as: working capital, investment loans, discounted bill, earmarked credit, and credit to finance international trade.

Table 1: Spreads (outstanding loans) and loan characteristics. The dependent variable is the $\log(1 + spread)$.

	(1)	(2)	(3)	(4)
Constant	0.4285***	0.2583***	0.2153***	0.1255***
	[41.77]	[23.15]	[27.45]	[13.21]
Maturity	-0.0025***	-0.0004***	-0.0004***	-0.0004***
	[-27.50]	[-17.57]	[-15.70]	[-16.03]
NPL	0.2973***	0.1604***	0.1372***	0.1367***
	[40.52]	[20.11]	[15.83]	[15.55]
NPL Lag (mean)	0.1259***	0.0639***	0.0789***	0.0758***
	[13.95]	[9.93]	[13.82]	[13.63]
NPL Lead (mean)	0.0894***	0.0228***	0.0491***	0.0488***
	[12.20]	[4.13]	[10.57]	[10.64]
Currency dummy	-0.0096**	-0.0061*	0.0026	0.0025
	[-2.59]	[-1.72]	[1.15]	[1.07]
Firm-bank Relationship	-0.0001***	-0.0001***	-0.0001***	-0.0001***
	[-25.86]	[-30.44]	[-29.91]	[-31.99]
Number of banks	-0.0433***	-0.0204***	-0.0162***	-0.0145***
	[-27.25]	[-18.25]	[-25.23]	[-21.22]
Observations	11,846,248	11,846,248	11,846,248	11,846,248
R-squared	0.121	0.454	0.487	0.502
Adj R-squared	0.121	0.454	0.487	0.495
Loan type controls	No	Yes	Yes	Yes
Sector FE	No	No	Yes	Yes
State FE	No	No	Yes	Yes
Firm-type FE	No	No	Yes	Yes
Time FE	No	No	Yes	Yes
Sector x State x Time FE	No	No	No	Yes
Earmarked dummy	No	No	No	Yes
RMSE	0.301	0.237	0.230	0.228

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Summary statistics of the dependent variable ($\log(1 + spread)$) and residuals of the regressions of Table 1. The number of observations are similar to the corresponding models of Table 1.

	log	Residuals			
	$(1 + spread)$	Model (1)	Model (2)	Model (3)	Model (4)
Mean	0.307	0.0000	0.0000	0.0000	0.0000
Median	0.211	-0.071	-0.019	-0.016	-0.014
St. Dev.	0.321	0.301	0.237	0.229	0.226
Min.	-0.108	-1.004	-1.213	-1.302	-1.289
P5	-0.031	-0.356	-0.343	-0.332	-0.330
P95	1.018	0.639	0.428	0.406	0.393
Max.	1.438	2.478	1.623	1.655	1.562
Skewness	1.51	1.36	0.85	0.87	0.85
Kurtosis	5.04	5.18	6.89	7.26	7.28

complete specification, firm fixed effects explain approximately 54 percent of the variability of the residual of the logarithm of one plus the spread rate. This suggests that about 2/3 of the observed variability of the logarithm of one plus the spread rate can be explained by loan characteristics, fixed effects of time, location and sector (regressors of Model (4) of Table 1) and firm fixed effects (regressors of Column (4) of Table 3).[5]

We finally regress the residuals of Column (4) of Table 3 on some time-varying observable firm characteristics, such as age, wage bill and the average years of education of its workers. The coefficients of such regression are reported on Column (1) of Table 4. Observe that, older and larger firms (measured by the wage bill) pay lower interest on loans. For instance, a firm that is one standard deviation larger (measured by the wage bill) would be paying -0.69% in the spread than an average firm. Firms with more educated workers, which might map into more productive firms, also pay lower loan

[5]Since firm fixed effects contain some of the fixed effects introduced in Model (4) of Table 1, we can also use Model (2) of Table 1 and Column (2) of Table 3 to reach similar conclusions.

Table 3: Spreads residuals of Table 1 and firm fixed-effects

Spreads residuals	(1)	(2)	(3)	(4)
R-squared	0.629	0.555	0.541	0.538
Adj. R-squared	0.499	0.398	0.380	0.375
RMSE	0.213	0.184	0.181	0.179
Observations	11,846,248	11,846,248	11,846,248	11,846,248
Firm FE	Yes	Yes	Yes	Yes
Firm FE St. Dev.	0.238	0.176	0.169	0.166

interest rates. Such relationships are statistically different from zero at 99% confidence level and it is important to highlight that such correlations hold after we control for a set of loan characteristics, time fixed effects and firm fixed effects. In Table 4 we also report the regression of the same residuals of Column (4) of Table 3 on the first principal component (PC) of the time-varying firm characteristics used in Column (1) of Table 4. The first principal component uses an orthogonal transformation of these observable variables, which are correlated, into one variable that contains most of the information of the regressors used in Column (1) of Table 4. We can observe that, for different specifications, the principal component of such variables is negatively correlated with the residuals of Column (4) of Table 3, confirming the negative correlation between age, size and productivity of the firm and the spread they pay on their loans.

We also implement analogous analysis by using the logarithm of the size of outstanding loans at the firm level as the dependent variable instead of the logarithm of one plus the spread rate on outstanding loans. We first regress the logarithm of loan size on a set of loan characteristics (e.g., loan maturity, type, among other characteristics) and on a set of fixed effects (e.g., time, location and activity) similar to those used in Table 1. The coefficients of such regressions are reported on Table 5. The coefficients show a negative correlation between the share of non-performing loan and the size of outstanding loans at the firm level. There is a positive relationship between loan maturity and loan size, as well as a positive correlation between the length of the firm-bank relationship and the loan size. In our most

Table 4: Spread residuals of Table 3 (Model (4)) and firm time varying observable characteristics

	Firm Characteristics	First Principal Components			
Residuals	(1)	(2)	(3)	(4)	(5)
Constant	0.0148***	0.0007***	-0.0036***	-0.0002***	-0.0014***
	[36.93]	[14.49]	[-32.72]	[-4.35]	[-21.32]
Age	-0.0003***				
	[-43.86]				
Age^2	0.0000***				
	[34.68]				
Wagebill (ln)	-0.0005***				
	[-14.91]				
Education	-0.0010***				
	[-22.85]				
PC		-0.0011***		-0.0042***	-0.0050***
		[-33.35]		[-44.10]	[-52.32]
PC (ln)			-0.0006***		
			[-9.84]		
PC^2				0.0005***	0.0033***
				[34.57]	[41.31]
PC^3					-0.0003***
					[-35.33]
Observations	10,850,982	10,850,982	3,031,317	10,850,982	10,850,982
R-squared	0.000	0.000	0.000	0.000	0.000
Adj. R-squared	0.000346	0.000105	2.99e-05	0.000226	0.000361
RMSE	0.155	0.155	0.168	0.155	0.155

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

complete specification, all regressors listed in Column (4) of Table 5 explain approximately 58% of the variability of loan size at the firm level.

In order to comprehend how the dispersion of spreads vary with firm loan characteristics, we then regress the outstanding loan residuals of all regressions reported in Table 5 on firm fixed effects. The summary of such regressions are reported on Table 6. Similarly to the conclusion of the spread regressions reported in Table 3, we can observe that about 2/3 of the observed loan variability is also explained by loan characteristics, time fixed effects and firm fixed effects (see Column (4) of Tables 5 and 6).

Then, finally, in Table 7 we regress the residuals of the regression of Model (4) of Table 6 on time-varying firm observable characteristics such as age, the logarithm of the wage bill and the average years of schooling of its employees. Loan outstanding at the firm level is positively correlated with firm age, size and the average year of schooling of its employees.The analysis using the first principal component of these three variables also confirms this positive association of age, size and productivity of the firm with the size outstanding loans at the firm level.

Therefore, this section shows that there is a large dispersion in the spread rate and in the loan size at the firm level. In addition, approximately 50 and 58 percent of the variability in the spread rate and in loan size, respectively, are explained by loan characteristics, time fixed effects and fixed effects of location and sector of economic activity. We also show that, even after we control for firm fixed effects, old, large and high productive firms not only borrow more but they also pay lower interest rates on their loans than young, small and less productive firms.

3 The Model

3.1 Environment

The economy is populated by a continuum of infinitely lived individuals. Time is discrete and infinite ($t = 0, 1, 2, ...$). There is one good that can be used for consumption or investment. Agents can be workers or entrepreneurs.

3.1.1 Endowments

In each period individuals are endowed with initial wealth, a, and they can be either a worker or an entrepreneur. Entrepreneurs create jobs and manage

Table 5: Outstanding loans and loan characteristics. The dependent variable is the log of outstanding loans

	(1)	(2)	(3)	(4)
Constant	9.2990***	12.0784***	12.2278***	11.9394***
	[150.37]	[125.40]	[143.81]	[33.00]
Maturity	0.0094***	0.0036***	0.0040***	0.0036***
	[22.14]	[22.65]	[26.01]	[26.87]
NPL	-0.9184***	-0.4489***	-0.4272***	-0.4174***
	[-24.43]	[-17.02]	[-13.41]	[-13.00]
NPL Lag (mean)	-1.0391***	-0.7417***	-0.8248***	-0.7211***
	[-21.37]	[-16.56]	[-22.32]	[-22.30]
NPL Lead (mean)	0.1330**	0.3040***	0.3525***	0.3612***
	[2.19]	[7.07]	[10.50]	[10.84]
Currency dummy	0.6153***	0.4747***	0.2976***	0.3039***
	[16.65]	[20.15]	[17.18]	[18.60]
Firm-bank Relationship	0.0003***	0.0002***	0.0002***	0.0002***
	[7.92]	[6.55]	[10.18]	[11.81]
Number of banks	0.7509***	0.6912***	0.6359***	0.5896***
	[69.17]	[55.58]	[56.83]	[64.31]
Observations	11,846,248	11,846,248	11,846,248	11,846,248
R-squared	0.422	0.505	0.561	0.582
Adj. R-squared	0.422	0.505	0.561	0.576
Loan type controls	No	Yes	Yes	Yes
Sector FE	No	No	Yes	Yes
State FE	No	No	Yes	Yes
Firm-type FE	No	No	Yes	Yes
Time FE	No	No	Yes	Yes
Sector x State x Time FE	No	No	No	Yes
Earmarked dummy	No	No	No	Yes
RMSE	1.358	1.257	1.184	1.163

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Outstanding loans residuals and firm fixed-effects

Outstd. loans residuals	(1)	(2)	(3)	(4)
R-squared	0.675	0.679	0.657	0.653
Adj. R-squared	0.561	0.567	0.537	0.532
RMSE	0.900	0.827	0.806	0.790
Observations	11,846,248	11,846,248	11,846,248	11,846,248
Firm FE	Yes	Yes	Yes	Yes
Firm FE St. Dev.	1.116	1.035	0.960	0.933

their labor force, n. Each individual is endowed with a talent for managing, z, drawn from an invariant Pareto distribution function $\mu(z) = \eta z^{-(\eta+1)}$ with $z \geq 1$. With probability $\gamma \in [0, 1]$ individuals keep the same talent from period t to period $t+1$, and with probability $1-\gamma$, individuals will draw a new talent for managing from distribution $\mu(z)$. Agents accumulate assets.

3.1.2 Preferences

Individuals derive utility from consumption, c_t, and preferences are represented by:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right], \tag{1}$$

where $\beta \in (0, 1)$ is the subjective discount factor, and E_0 is the expectations operator conditional on information at $t = 0$. The period utility is assumed to take the following form:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \ \sigma > 0. \tag{2}$$

3.1.3 Technology

Managers in the entrepreneurial sector operate a technology that uses labor, n, and capital, k, to produce a single consumption good, y, that is represented by

$$y = f(z; k, n) = z k^{\alpha} n^{\theta}, \ \theta, \ \alpha \in (0, 1), \quad \text{and} \quad \alpha + \theta < 1. \tag{3}$$

Table 7: Outstanding loans residuals (Model 4) and firm productivity

	Firm Characteristics	First Principal Components			
Residuals	(1)	(2)	(3)	(4)	(5)
Constant	-0.2503***	-0.0021***	0.0357***	0.0118***	0.0277***
	[-137.03]	[-10.19]	[68.32]	[45.19]	[94.25]
Age	0.0030***				
	[90.72]				
Age^2	-0.0000***				
	[-74.65]				
Wagebill (ln)	0.0179***				
	[105.17]				
Education	0.0103***				
	[55.22]				
PC		0.0120***		0.0579***	0.0694***
		[75.63]		[130.85]	[159.16]
PC (ln)			0.0002		
			[0.70]		
PC^2				-0.0071***	-0.0461***
				[-110.03]	[-125.10]
PC^3					0.0048***
					[103.58]
Observations	10,850,982	10,850,982	3,031,317	10,850,982	10,850,982
R-squared	0.003	0.001	0.000	0.002	0.003
Adj. R-squared	0.00331	0.000609	-1.77e-07	0.00200	0.00335
RMSE	0.683	0.683	0.773	0.683	0.683

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Managers can operate only one project. Entrepreneurs finance part of their capital through their own assets, and part by borrowing from financial intermediaries. Entrepreneurs incur a fixed-cost κ to operate in every period.

Entrepreneurs face financial frictions. Agents have two options in which to invest their assets:

- Financial Intermediaries: Agents can competitively rent capital to financial intermediaries (banks) and earn an endogenously determined interest rate, r.

- Own business: Agents can use their own wealth as part of the amount required to operate a business. They might borrow the remaining capital they require from a bank at interest rate $\tilde{r}$.

3.2 Financial intermediaries

There is a continuum of financial intermediaries who are competing to lend to each entrepreneur. The cost of raising capital is r. There is an intermediation cost associated with making a loan $l = k - a$ for an entrepreneur who has collateral a. We denote this cost by

$$g(l, a) = \tau_0 l + l\left(\frac{\tau_a}{a} + \frac{\tau_z}{z}\right), \quad \text{with} \quad \tau_a \geq 0 \text{ and } \tau_z \geq 0.$$

We assume that τ_0 can be idiosyncratic to each entrepreneur and let τ_0 be identically and independently drawn from an Uniform distribution with support $[0, \bar{\tau}_0]$.

If the lender does not incur this cost, the borrower could break the financial contract without any cost; otherwise, borrowers will incur a cost, which is described below.

Since lenders compete for each loan, then:

$$\tilde{r}l = rl + \tau_0 l + l\left(\frac{\tau_a}{a} + \frac{\tau_z}{z}\right) \implies \tilde{r}(a, z) = r + \tau_0 + \frac{\tau_a}{a} + \frac{\tau_z}{z}. \tag{4}$$

The spread between the loan rate $\tilde{r}(a, z, \tau_0)$ and the deposit rate would be decreasing in the level of collateral, a, and on the quality of the entrepreneur's project.

There is a commitment and limited liability problem in the credit market. Borrowers cannot commit *ex-ante* to repay. Those who default on their debt incur a cost ϕ proportional to the output produced net of labor costs.

Agents with sufficient resources and managerial ability to become entrepreneurs choose the level of capital and number of employees to maximize profit subject to a technological constraint and a credit market incentive constraint, i.e.,

$$\pi(a, z, \tau_0) = \max_{n,k \geq 0} zk^\alpha n^\theta - wn - \tilde{r}(a, z, \tau_0)(k - a) - ra - \kappa, \tag{5}$$

subject to the credit market incentive constraint

$$\begin{aligned} zk^\alpha n^\theta - wn - \tilde{r}(a, z, \tau_0)(k - a) - ra - \kappa \geq \\ (1 - \phi)(zk^\alpha n^\theta - wn) - ra - \kappa. \end{aligned} \tag{6}$$

We can then rewrite the incentive compatibility constraint as:

$$k \leq a + \frac{\phi(zk^\alpha n^\theta - wn)}{\tilde{r}(a, z, \tau_0)}. \tag{7}$$

Both the loan size and the loan interest rate are jointly determined, which is a fact emphasized by Banerjee (2003). They are outcome variables which are jointly determined by primitive variables, such as the net worth of the borrower, as well as her productivity, the intermediation cost technology, and the financial friction related to a commitment and limited liability problem.

3.3 Optimal Decisions

Let $V^w(a, z, \tau_0)$ and $V^e(a, z, \tau_0)$ be the value for individual (a, z, τ_0) to become a worker or an entrepreneur, respectively. The occupational choice of an individual (a, z, τ_0) is described by the following value function

$$V(a, z, \tau_0) = \max\{V^w(a, z, \tau_0), V^e(a, z, \tau_0)\}. \tag{8}$$

This defines the policy function $o(a, z, \tau_0)$ such that $o(a, z, \tau_0) = 1$ if the individual becomes an entrepreneur and zero otherwise. The value function of being a worker is defined by the following Bellman equation:

$$V^w(a, z, \tau_0) = \max_{c,a' \geq 0} \{u(c) + \beta E_{z',\tau_0'}[V(a', z', \tau_0')|z, \tau_0]\}, \tag{9}$$

subject to

$$c + a' = w + (1 + r - \delta)a. \tag{10}$$

Analogously, the value of becoming an entrepreneur is given by:

$$V^e(a, z, \tau_0) = \max_{c, a' \geq 0} \{u(c) + \beta E_{z', \tau_0'}[V(a', z', \tau_0')|z, \tau_0]\}, \tag{11}$$

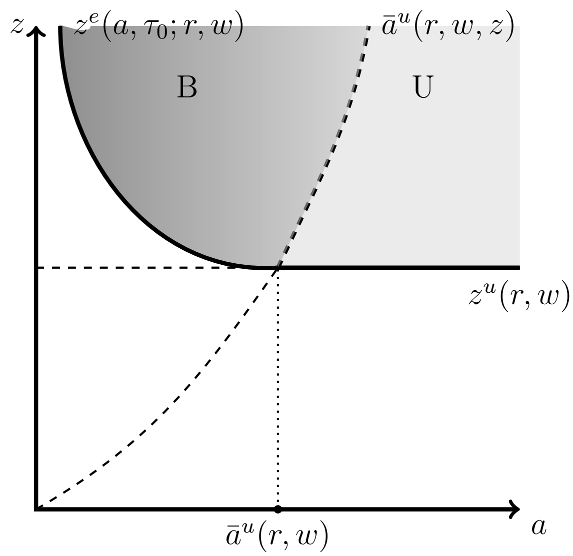
subject to

$$c + a' = \pi(a, z, \tau_0) + (1 + r - \delta)a. \tag{12}$$

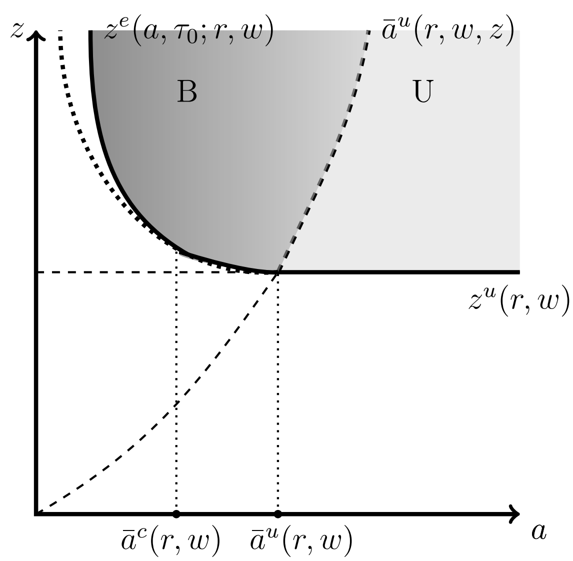
Figure 1 shows occupational choice in (z, a) space for this economy for a given τ_0, wage rate and interest rate r and two levels of the enforcement variable ϕ. Appendix B contains the formal proof of the two graphs presented in Figure 1. Figure 1 (a) shows the case of perfect enforcement, such that $\phi = 1$, but in which financial intermediation is still costly and there exists a positive spread rate $\tilde{r}(a, z, \tau_0) - r > 0$. The light gray shaded area (region U) displays the measure of agents who are unconstrained entrepreneurs, such that their initial wealth a is more than sufficient to operate a business such that their optimal capital stock is below their initial wealth, $k^u(z, r, w) \leq a$. Agents with entrepreneurial ability and wealth lying in this region produce at their optimal scale, and they all have the same marginal productivity of rented inputs employed in production. Among these entrepreneurs there is no misallocation of capital. Without any financial friction, all agents represented by a pair (z, a) lying above the dotted line $z^u(r, w)$ would become an entrepreneur producing at their optimal scale and with the same marginal productivity of inputs. But with costly intermediation this is not the case. The line defining the occupational choice is represented by the solid line $z^e(a, \tau_0; r, w)$. The dark gray shaded area (region B) represents the entrepreneurs who are borrowers. They are paying different loan interest rates, depending on their initial net worth a and productivity z - we are assuming a given τ_0. Consequently, they produce at very different levels of marginal productivity of capital. Agents with a pair (z, a) close to the dotted line $\bar{a}^u(r, w, z)$ are borrowing less and have a marginal productivity of capital that is close to the rental price of capital. Agents far from this line are borrowing more and producing at a higher marginal productivity of capital. Notice that intermediation costs affect not only the misallocation of capital at the intensive margin, but they also affect the allocation of talent since agents with a pair (z, a) such that $z^u(r, w) \leq z \leq z^e(a, \tau_0; r, w)$ do not become entrepreneurs due to such costs.

Figure 1 (b) displays the case in which the enforcement of financial contracts are imperfect, such that $\phi < 1$. Region U still represents the measure

Figure 1: Light gray shaded area shows the measure of agents who are unconstrained entrepreneurs. Dark gray shaded area shows the measure of agents who are not constrained borrowers. The white area below the curve $z^e(a; r, w)$ represents the measure of agents who are workers.



(a) Full enforcement, $\phi = 1$.



(b) Imperfect enforcement, $\phi < 1$.

of unconstrained entrepreneurs. Region B corresponds to entrepreneurs who are borrowers. For a given τ_0, Agents with a pair (z, a) in region B which is close to the dotted line $\bar{a}^u(z; r, w)$ are unconstrained borrowers such that the incentive compatible constraint of financial contracts is not biding. Such agents produce with a marginal productivity of capital similar to the loan rate, which varies with their initial net worth and productivity. As we get closer to the solid line $z^e(a, \tau_0; r, w)$, then the incentive compatible constraint binds and entrepreneurs will be producing with a marginal productivity of capital that is above the loan rate they face. Imperfect enforcement of financial contracts affect also the allocation of talent, since the line $z^e(a, \tau_0; r, w)$ becomes steeper when this constraint starts to bind at $\bar{a}^c(r, w)$, and the measure of entrepreneurs decrease.

4 Quantitative Analysis

In order to solve the model numerically, we must assign values to the model parameters. The model period is set to 1 year. There are eleven structural parameters to discipline. A full calibration targeting Brazilian data is currently being implemented. For now, the parameters are set to usual values and we take values as close as possible to Buera, Kaboski, and Shin (2011).

- **Preferences:** the discount factor β is set to 0.92 and the coefficient of relative risk aversion σ to 1.5.

- **Stochastic process:** the distribution of productivity shocks is characterized by two parameters: γ and η. These are taken from Buera, Kaboski, and Shin (2011) such that $\gamma = 0.85$ and $\eta = 4.84$.

- **Technology:** $\alpha + \theta = 0.79$ with $\alpha = 0.3 \times (1/4.84 + 0.79)$ - Capital share is $\frac{\alpha}{\frac{1}{\eta}+\alpha+\theta}$. Fixed cost $\kappa = 4$.

- **Financial sector:** the monitoring cost function has three parameters: $\bar{\tau}_0$, which controls the mean spread; τ_a, which determines how the spread varies with borrowers' assets; and τ_z, which defines how the entrepreneurial productivity affects interest on loans. Now, we set $\bar{\tau}_0$ and τ_z to 0 in the benchmark. The parameter τ_a will be a free parameter in the benchmark exercise. The enforcement constraint is characterized by the parameter ϕ, which will be another free parameter.

Table 8: Benchmark ($\tau_z = 0.15$ and $\phi = 0.10$) and Improved Financial Sector

	Benchmark	$\tau_a = 0$	$\phi \uparrow 0.27$
% Entrepreneurs	25.8	29.3	19.2
GDP	1.00	1.25	1.34
Capital	1.00	2.11	2.08
TFP	1.00	0.97	1.05
Credit/GDP	0.542	1.99	1.98
Average Spread	3.41	0.0	3.46
Max Spread	5.12	0.0	5.44
Min Spread	3.0	0.0	3.0
Average Leverage	0.44	2.32	2.88
Average Firm Size	2.80	2.33	4.34
Max/Min Firm Size	2.83	1.97	2.28
Wage	1.00	1.32	1.16

The wage w is determined to clear the labor market in equilibrium. As for the interest rate r, we assume that this is a small open economy and we set $r = 0.02$.

Selected moments for the benchmark economy are reported in Table 8. Approximately one quarter of the individuals in the economy consists of entrepreneurs. The Credit/GDP ratio is 0.54. The average spread paid by entrepreneurs that borrow is 3.41%. Some entrepreneurs, however, pay considerably more for their credit as the maximum spread is higher than 5%. The average leverage ratio stands at 0.44. In this example economy, firms are heterogenous in size, but modestly so: the largest firm has 2.8 times more employees than the smallest firm.

Now, consider running the following counterfactual experiment. Suppose there is no monitoring costs to financial intermediaries. To implement this, set $\phi_a = 0$. In this world, the interest rate for credit operation will equal the deposit rate and the spread will thus be equal to 0. The results for this economy are reported in the second column of Table 8. First, note that GDP increases by 25%. More individuals choose to be entrepreneurs and their share rises to 29.3%. Without monitoring costs, credit is cheaper for all those who choose to be entrepreneurs, including those with low wealth. With cheaper financing, the Credit/GDP ratio increases to almost 2, which represents approximately 3.7 times the benchmark value. Accordingly, leverage increases 5-fold. Notice that capital, however, increases by approximately 100% relative to the baseline. There is less self-financing and agents borrow more to accumulate capital. Average firm size declines somewhat as cheaper credit helps smaller firms more. Large firms tend to have a high level of assets and thus already paid low spreads in the benchmark economy. Interestingly, TFP decreases by 3%.

Another important parameter affecting credit in this economy is the enforcement parameter ϕ, which controls the collateral constraint. Now, we increase the enforcement technology such that ϕ rises. To make the experiments comparable, we engineer a rise in ϕ such that the Credit/GDP ratio is the same as in the zero-spread economy: 1.98. The results for this experiment are reported in the third column of Table 8. First, GDP increases by even more than in the previous exercise and is now 34% higher than in the benchmark economy. One difference is that the share of entrepreneurs decreases. The average spread increases slightly and the maximum spread also rises. This experiment helps particularly high productivity agents that have low level of assets and are credit constrained. The average size of firms increase relatively to the benchmark, as well as TFP, which increases by 5%.

One more issue to report regarding both counterfactual exercises in Table 8 is that the wage increases relative to the benchmark. As financing is cheaper in both alternative scenarios, with a constant wage, more agents would choose to become entrepreneurs and labor demand increases due to more capital employed. In order to guarantee that the labor market clears, then the wage rate has to rise.

Another alternative scenario is to consider a world in which the spread is positive, but is constant across all borrowers. In order to implement this, set $\tau_a = 0$ and $\tau_0 > 0$ constant. In order to make this counterfactual comparable to the benchmark, set $\tau_0 = 0.0341$, i.e., the average spread in the benchmark economy. That is, all borrowers will pay a spread of 3.41%, regardless of their leverage. The results for this experiment are reported in the second column of Table 9. GDP is now 4% higher than in the economy with dispersion in financing costs. Note that the share of entrepreneurs increases slightly and so does the Credit/GDP ratio. Though the average spread is the same, by construction, firms that were paying higher prices for credit now have access to cheaper financing. This is particularly important for high productivity and low asset agents. Leverage then goes up.

In the previous scenario, though the spreads were kept at the same level as in the benchmark economy, credit was more abundant in the economy. An

Table 9: Benchmark and Constant (Fixed) Spread

	Benchmark	$\tau_a = 0$ and $\tau_0 > 0$
% Entrepreneurs	25.8	27.3
GDP	1.00	1.04
Capital	1.00	1.10
TFP	1.00	1.01
Credit/GDP	0.542	0.74
Average Spread	3.41	3.41
Max Spread	5.12	3.41
Min Spread	3.0	3.41
Average Leverage	0.44	0.69
Average Firm Size	2.80	2.61
Max/Min Firm Size	2.83	2.00
Wage	1.00	1.04

alternative counterfactual is to set a constant level for the spread that is consistent with the same Credit/GDP ratio as in the benchmark economy. This is done in Table 10. First note that output declines slightly. This happens because credit is more expensive now. Note, however, that the maximum spread paid in this economy is still a bit lower than in the benchmark. This leads some high productivity and low asset agents to become entrepreneurs, which leads their share to increase slightly. Again, in order to guarantee that the labor market clears, the wage rate rises.

We can also improve the enforcement technology using this economy as a new benchmark. This is done in Table 11. For comparison purposes, we engineer a hike in η to generate the same rise in the Credit/GDP ratio found in the similar exercise reported in Table 8. The parameter η had to be increased further in this experiment in order to generate the same rise in Credit/GDP. In this experiment, output rises (but less so than in the experiment with dispersion in financing costs). The share of entrepreneurs declines, but less so than in the experiments with varying spreads. Therefore, abstracting from dispersion in financing costs can underestimate the aggregate effects of financial reforms, which increase credit rights.

Table 10: Benchmark and Constant (Higher) Spread

	Benchmark	$\tau_a = 0$ and $\tau_0 > 0$
% Entrepreneurs	25.8	26.7
GDP	1.00	0.98
Capital	1.00	0.96
TFP	1.00	0.99
Credit/GDP	0.542	0.543
Average Spread	3.41	4.61
Max Spread	5.12	4.61
Min Spread	3.0	4.61
Average Leverage	0.44	0.50
Average Firm Size	2.80	2.60
Max/Min Firm Size	2.83	2.24
Wage	1.00	1.02

Table 11: Constant Spread and Better Enforcement

	$\tau_a = 0$ and $\tau_0 > 0$	Higher ϕ 0.33
% Entrepreneurs	26.7	22.6
GDP	0.98	1.27
Capital	1.00	2.02
TFP	1.00	1.03
Credit/GDP	0.543	1.98
Average Spread	4.61	4.61
Max Spread	4.61	4.61
Min Spread	4.61	4.61
Average Leverage	0.50	3.62
Average Firm Size	2.60	3.33
Max/Min Firm Size	2.24	2.01
Wage	1.02	1.19

5 Conclusion

In this paper, we investigate the effects of dispersion in the cost of financial intermediation on entrepreneurship, firm dynamics and productivity in an economy in which financial contracts are imperfectly enforced. We calibrate and estimate the model to be consistent with key firm level characteristics of the Brazilian economy, such as firm size, exit and entry rates. We then study how financial frictions affect firm dynamics, such as firm growth, entry and exit. In addition, we investigate the quantitative aggregate effects of the two financial frictions. We show that they produce very different aggregate effects. Enforcement of financial contracts has a larger aggregate impact on output (for a similar increase in the level of credit) than intermediation costs and improve total factor productivity. This is because when the enforcement of financial contracts increases then entrepreneurs can borrow more for a given interest rate, and this affects mainly more productive entrepreneurs who are credit constrained and can now grow faster. When intermediation costs decrease, then this affects all entrepreneur who are borrowing and those who can now borrow at a lower rate. This also increases production but the credit is not mainly allocated to the most productive entrepreneurs who are constrained. In this case TFP decreases. We also show that abstracting from dispersion in financing costs can underestimate the aggregate effects of reforms which improve credit rights.

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A Data Appendix

The definitions and source for the variables used in Section **??** are described below: TBA

Table 12: Summary Statistics

	Obs	Mean	Median	St. Dev.	Min.	P5	P95	Max.	Skewness	Kurtosis
Outst. loan	11,846,248	909,708	60,414	6.92e+07	0.001	3,781	1,093,925	1.12e+11	957.19	1,291,393
Spread	11,846,248	44.43	23.47	60.41	-10.23	-3.05	176.74	321.28	2.46	9.61
Maturity										
NPL	11,846,248	0.043	0.000	0.122	0	0	0.253	1	4.62	28.35
Currency dum.	11,846,248	0.037	0	0.190	0	0	0	1	4.86	24.61
Firm-bank rel.	11,846,248	80.16	45.55	120.67	0	1.49	246.98	1,407	4.61	32.69
N. of banks	11,846,248	2.12	2	1.415	1	1	5	36	2.37	14.64
Size	11,846,248	21.03	4	279.10	0	0	49	105,455	128.65	27,350
Age	11,846,236	14.48	9.24	19.29	0	1.52	38.50	118.62	3.88	20.10
Wage bill	11,846,248	33,798	3,505	989,213	0	0	61,411	1.10e+09	599.93	562,160
Education	10,895,213	6.32	6.62	1.12	-1	4.14	7.89	11	-0.80	4.30
Got loan	11,846,248	0.659	1	0	0	1	1	-0.67	1.45	
Firm growth	9,167,466	0.010	0	0.534	-11.20	-0.693	0.754	8.18		
Exit	11,284,288	0.082	0	0.274	0	0	1	1	3.04	10.28

B Mathematical Appendix

Consider the problem of an entrepreneur (a, z, τ_0). Let $d \leq a$ be the amount of assets entrepreneurs use in their business, and let l be loans, such that $k = d + l$. Clearly, since $\tilde{r} > r$ for all finite a, then if $l > 0$, then $d = a$. The problem of the entrepreneur can be rewritten as

$$\pi(a, z, \tau_0) = \max_{n,d,l \geq 0} z(d+l)^\alpha n^\theta - wn - \tilde{r}l - rd - \kappa, \tag{13}$$

subject to

$$l \leq \frac{\phi(z(d+l)^\alpha n^\theta - wn)}{\tilde{r}}, \quad \text{with } \tilde{r} = r + \tau_0 + \frac{\tau_a}{a} + \frac{\tau_z}{z}, \tag{14}$$

$$d \leq a. \tag{15}$$

The Lagrangean associated to this problem is:

$$L = z(d+l)^\alpha n^\theta - wn - \tilde{r}l - rd - \chi + \lambda \left[\frac{\phi(z(d+l)^\alpha n^\theta - wn)}{\tilde{r}} - l \right] + \mu[a-d]$$

The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial n} = \left(\theta \frac{y}{n} - w\right)\left(1 + \lambda \frac{\phi}{\tilde{r}}\right) \leq 0,\ n \geq 0,\ \frac{\partial L}{\partial n} n = 0, \tag{16}$$

$$\frac{\partial L}{\partial d} = \alpha \frac{y}{k}\left(1 + \lambda \frac{\phi}{\tilde{r}}\right) - r - \mu \leq 0,\ d \geq 0,\ \frac{\partial L}{\partial d} d = 0, \tag{17}$$

$$\frac{\partial L}{\partial l} = \alpha \frac{y}{k}\left(1 + \lambda \frac{\phi}{\tilde{r}}\right) - \tilde{r} - \lambda \leq 0,\ l \geq 0,\ \frac{\partial L}{\partial l} l = 0, \tag{18}$$

$$\mu[a-d] = 0, \tag{19}$$

$$\lambda \left[\frac{\phi(z(d+l)^\alpha n^\theta - wn)}{\tilde{r}} - l \right] = 0. \tag{20}$$

Case 1: If $0 < d < a$, then $\mu = 0$ and $\lambda = 0$. Therefore:

$$\theta \frac{y}{n} = w,\ \alpha \frac{y}{k} = r, \quad \text{and} \quad \alpha \frac{y}{k} < \tilde{r}.$$

It can be shown that

$$k^u(r, w; z) = \left(z \left(\frac{\alpha}{r}\right)^{(1-\theta)} \left(\frac{\theta}{w}\right)^\theta \right)^{\frac{1}{1-\alpha-\theta}},$$

$$n^u(r,w;z) = \left(z \left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\theta}{w}\right)^{1-\alpha} \right)^{\frac{1}{1-\alpha-\theta}},$$

$$y^u(r,w;z) = \left(z \left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\theta}{w}\right)^{\theta} \right)^{\frac{1}{1-\alpha-\theta}},$$

and

$$\pi^u(r,w;z) = (1-\alpha-\theta) y^u(r,w;z) - \kappa.$$

Therefore, $\pi^u(r,w;z) \geq w$ defines a threshold ability level $z^u(r,w)$ given by

$$z^u(r,w) = \left(\frac{w+\kappa}{1-\alpha-\theta}\right)^{1-\alpha-\theta} \left(\frac{r}{\alpha}\right)^{\alpha} \left(\frac{w}{\theta}\right)^{\theta},$$

such that for all agents (a,z) with $a > k^u(r,w;z)$, and $z \geq z^u(r,w)$ agents are entrepreneurs. Notice that $z^u(r,w)$ is independent of a and τ_0. Since $k^u(r,w;z)$ is increasing with z, we can define a threshold of wealth $\bar{a}^u(r,w,z)$ and all agents with $z > z^u(r,w)$ and $a > \bar{a}^u(r,w,z)$ are unconstrained entrepreneurs.

Case 2: If $d = a > 0$ and $l = 0$, then $\mu > 0$ and $\lambda = 0$. Consequently:

$$\theta \frac{y}{n} = w, \ \ \alpha \frac{y}{k} = r + \mu, \quad \text{and} \quad \alpha \frac{y}{k} < \tilde{r}.$$

It can be shown that

$$k^{nb}(r,w;z,a,\tau_0) = a,$$

$$n^{nb}(r,w;z,a,\tau_0) = \left(z a^{\alpha} \left(\frac{\theta}{w}\right) \right)^{\frac{1}{1-\theta}},$$

$$y^{nb}(r,w;z,a,\tau_0) = \left(z a^{\alpha} \left(\frac{\theta}{w}\right)^{\theta} \right)^{\frac{1}{1-\theta}},$$

and

$$\pi^{nb}(r,w;z,a,\tau_0) = (1-\theta) y^{nb}(r,w;z,a,\tau_0) - ra - \kappa.$$

Condition $\pi^{nb}(r,w;z,a,\tau_0) \geq w$ defines a treshold ability level $z^{nb}(r,w;z,a,\tau_0)$ given by

$$z^{nb}(r,w,a,\tau_0) = \left(\frac{w+\kappa+ra}{1-\theta}\right)^{1-\theta} \left(\frac{w}{\theta}\right)^{\theta} \frac{1}{a^{\alpha}},$$

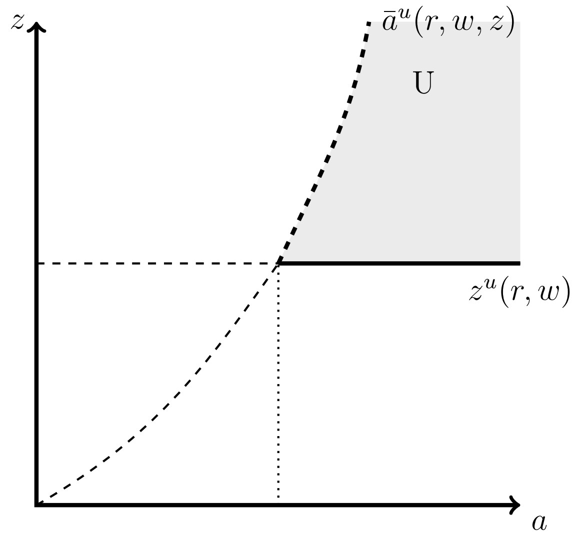

Figure 2: Case 1: $k^u(r,w) \leq a$. Light gray shaded area shows the measure of agents who are unconstrained entrepreneurs.

such that for all agents with $k^{nb}(r,w;z,a,\tau_0) = a$, and $z \geq z^{nb}(r,w,a,\tau_0)$ agents are entrepreneurs. Observe that since $1-\alpha-\theta > 0$, then $\lim_{a\to 0} z^{nb}(r,w,a,\tau_0) = \infty$. It can be shown that

$$sign\left(\frac{\partial z^{nb}(r,w,a,\tau_0)}{\partial a}\right) = sign\left((1-\alpha-\theta)ra - \alpha(w+\kappa)\right).$$

Notice that since $\alpha\frac{y}{a} = r+\mu$ at $z^{nb}(r,w,a,\tau_0)$, such that $(1-\theta)y^{nb}(r,w;z,a,\tau_0) - ra - \kappa = w$, we have

$$sign\left(\frac{\partial z^{nb}(r,w,a,\tau_0)}{\partial a}\right) = sign\left(-\mu a\right).$$

This is clearly negative, as long as $\mu > 0$. Therefore as $a \to \bar{a}^u(r,w)$, then $z^{nb}(r,w,a) \to z^u(r,w)$.

Case 3: If $d = a > 0$ and $0 < l < \frac{\phi(z(d+l)^\alpha n^\theta - wn)}{\tilde{r}}$, then $\mu > 0$ and $\lambda = 0$. Consequently:

$$\theta\frac{y}{n} = w,\ \ \alpha\frac{y}{k} = r+\mu, \ \text{ and } \ \alpha\frac{y}{k} = \tilde{r}.$$

It can be shown that

$$k^b(\tilde{r}, w; z) = \left(z \left(\frac{\alpha}{\tilde{r}} \right)^{(1-\theta)} \left(\frac{\theta}{w} \right)^{\theta} \right)^{\frac{1}{1-\alpha-\theta}},$$

$$n^b(\tilde{r}, w; z) = \left(z \left(\frac{\alpha}{\tilde{r}} \right)^{\alpha} \left(\frac{\theta}{w} \right)^{1-\alpha} \right)^{\frac{1}{1-\alpha-\theta}},$$

$$y^b(\tilde{r}, w; z) = \left(z \left(\frac{\alpha}{\tilde{r}} \right)^{\alpha} \left(\frac{\theta}{w} \right)^{\theta} \right)^{\frac{1}{1-\alpha-\theta}},$$

and

$$\pi^b(\tilde{r}, w; z) = (1-\alpha-\theta) y^b(\tilde{r}, w; z) + (\tilde{r} - r)a - \kappa.$$

Therefore, given that $\tilde{r} = r + \tau_0 + \frac{\tau_a}{a} + \frac{\tau_z}{z}$, the inequality $\pi^b(\tilde{r}, w; z) \geq w$ defines an ability level $z^b(\tilde{r}, w; a)$ given by

$$z^b(\tilde{r}, w, a) = \left(\frac{w + \kappa - (\tilde{r} - r)a}{1-\alpha-\theta} \right)^{1-\alpha-\theta} \left(\frac{r}{\alpha} \right)^{\alpha} \left(\frac{w}{\theta} \right)^{\theta},$$

such that for all agents with $a < k^b(\tilde{r}, w; z) < \frac{\phi(z(d+l)^{\alpha} n^{\theta} - wn)}{\tilde{r}}$, and $z \geq z^b(\tilde{r}, w, a)$, then agents are entrepreneurs. Observe that $\frac{\partial z^b(\tilde{r}, w, a)}{\partial a} < 0$, and $\lim_{a \to 0} z^b(\tilde{r}, w, a) = \infty$.

Cases 2 and 3 imply that for all $a \in [0, \bar{a}^u(r, w)]$, there will be a productivity level $z^e(\tilde{r}, w, a) = \max\{z^u(r, w), \min(z^{nb}(\tilde{r}, w, a), z^b(\tilde{r}, w, a))\}$ such that $z^e(\tilde{r}, w, \bar{a}^u(r, w)) = z^u(r, w)$, $\frac{\partial z^e(\tilde{r}, w, a)}{\partial a} < 0$, and $\lim_{a \to 0} z^b(\tilde{r}, w, a) = \infty$. In addition, whenever $z \geq z^e(\tilde{r}, w, \bar{a}^u(r, w))$, then the agent is an entrepreneur.

Case 4: If $d = a > 0$ and $l = \frac{\phi(z(a+l)^{\alpha} n^{\theta} - wn)}{\tilde{r}}$, then $\mu > 0$ and $\lambda > 0$. Consequently:

$$\theta \frac{y}{n} = w, \ \alpha \frac{y}{k} \left(1 + \lambda \frac{\phi}{\tilde{r}} \right) = r + \mu, \quad \text{and} \quad \alpha \frac{y}{k} \left(1 + \lambda \frac{\phi}{\tilde{r}} \right) = \tilde{r} + \lambda.$$

Given that the amount of capital is constrained, it must be the case that $\alpha \frac{y}{k} > \tilde{r}$. The labor first-order condition yields:

$$n(w; k^c, z) = \left(z (k^c))^{\alpha} \left(\frac{\theta}{w} \right) \right)^{\frac{1}{1-\theta}},$$

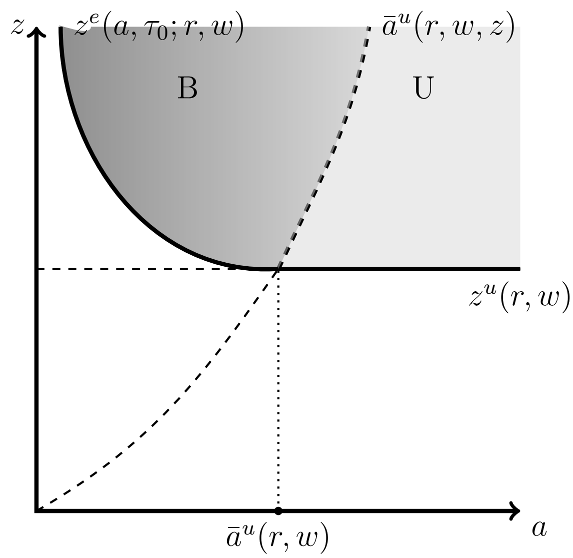

Figure 3: Cases 2 and 3: $k^u(r, w) < a$ and $0 \leq k - a < \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$. Light gray shaded area shows the measure of agents who are unconstrained entrepreneurs. Dark gray shaded area shows the measure of agents who are not constrained borrowers.

where k^c solves

$$k^c = a + \frac{\phi(z(k^c)^\alpha n(w;k^c,z)^\theta - wn(w;k^c,z))}{\tilde{r}}.$$

This equation defines

$$k^c = k^c(\tilde{r},w;z,a), \quad \text{with} \ \frac{\partial k^c}{\partial a} > 0, \ \frac{\partial k^c}{\partial z} > 0.$$

The derivatives can be checked using the Implicit Function Theorem. We have that

$$y^c(\tilde{r},w;z,a) = \left(zk^c(\tilde{r},w;z,a)^\alpha \left(\frac{\theta}{w}\right)^\theta \right)^{\frac{1}{1-\theta}}$$

and

$$\pi^c(\tilde{r},w;z,a) = (1-\theta)(1-\phi)y^c(\tilde{r},w;z,a) - ra - \kappa.$$

Condition $\pi^c(\tilde{r},w;z,a) \geq w$ defines a threshold ability level $\bar{z}^c(\tilde{r},w;a)$, which is decreasing in a as long as $\lambda > 0$. We can show that $\lim_{a\to 0} \bar{z}^c(\tilde{r},w;a) = \infty$. Observe that when $\lambda = 0$ and $l = \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$, then for agents who are indifferent to be entrepreneurs or workers, we have that $\bar{z}^c(\tilde{r},w;a) = \bar{z}^b(\tilde{r},w;a)$. This defines a value $\bar{a}^c(w,\tilde{r})$, such that whenever $a < \bar{a}^c(w,\tilde{r})$ and $\bar{z}^b(\tilde{r},w;a) \leq z \leq \bar{z}^c(\tilde{r},w;a)$, the leverage constraint is binding. For such agents, then $l = \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$ and $\lambda > 0$, and $\bar{z}^c(\tilde{r},w;a) > \bar{z}^b(\tilde{r},w;a)$, in order to compensate for the low capital used. Therefore for any $\bar{z}^b(\tilde{r},w;a) \leq z \leq \bar{z}^c(\tilde{r},w;a)$ and $a < \bar{a}^c(w,\tilde{r})$, the occupational choice is restricted by the leverage ratio. This is shown in the figure below.

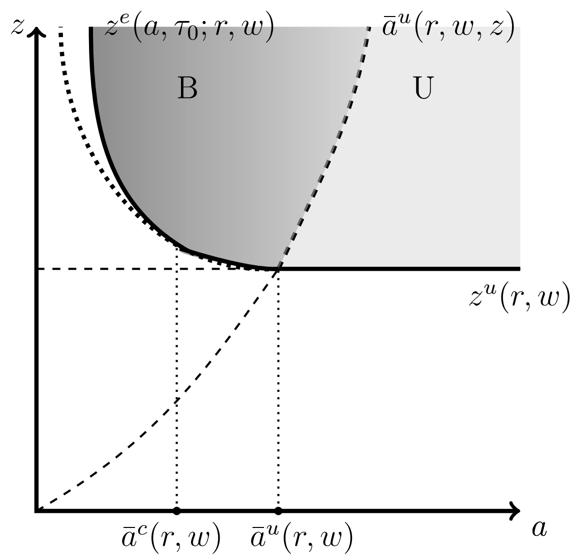

Figure 4: Cases 4: $k^u(r, w) < a$ and $k - a = \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$. Light gray shaded area shows the measure of agents who are unconstrained entrepreneurs. Dark gray shaded area shows the measure of agents who are not constrained borrowers.